ISSN 2229-5518

Vibration reduction, stability and resonance of a dynamical system excited by external and parametric excitations via time-delay absorber

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Abstract—Vibrations and dynamic chaos are undesired phenomenon in structures as they cause the 4D. They are: disturbance, discomfort, damage and destruction of the system or the structure. For these reasons, money, time and effort are spent to eliminate or control vibrations, noise and chaos or to minimize them. Vibration control is classified into two main categories: passive control and active control. In this paper, the analytical solution of the nonlinear dynamical system using multiple time scale method up to and including second order approximations are obtained. All resonance cases from analytical solution are extracted. The numerical solution of the non-linear dynamical system using Runge-Kutta method of order four are obtained. The stability of the dynamical system at the worst resonance case is studied. The behaviors of the system at different values of excitations are investigated. The effects of various parameters on the behavior of the system are studied. Comparison with the available published work is reported.

Index Terms— Vibration, Resonance, Time-delay, Stability.

1 Introduction

Vibrations and dynamic chaos are undesired phenomenon in structures as they cause the 4D. They are: disturbance, discomfort, damage and destruction of the system or the structure. For these reasons, money, time and effort are spent to eliminate or control vibrations, noise and chaos or to minimize them. Structures and mechanical systems should be designed to enable better performance under different types of loading, particularly dynamic and transient loads. Vibration control is classified into two main categories: passive control and active control. The study of vibrating structures has been a subject of a particular interest in recent years. This is due to the fact that structures under multi-parametric excitation forces appear in various fields of fundamental and applied sciences [1-2]. Another way to control the bending vibration of the beam structure, is to couple it in a sandwich manner to a linear beam-type dynamic vibration absorber as in ref [3- 4]. In ref [5] the authors used the nonlinearity of a foundation and showed that the behavior of the beam could be expressed by a ϕ^6 potential. Another important center of interest is the study of vibrating structures under active control [6-9]. In ref [10], the authors considered such a problem in linear structures and showed that time-delay can even lead to the instability of the whole structure. Nana et al. [11-13] studied the modeling and optimal active control with time delay dynamics of a strongly nonlinear beam. The control by sandwich beam and the one using piezoelectric absorber are investigated. El-Ganaini and Elgohary studied the vibration of a damped buckled beam subject to multi-external excitation [14] and multi-parametric excitation forces [15]. The model is represented by two-degree-of-freedom system consisting of the main system and the absorber. The stability of the system is investigated numerically applying both phase-plane and frequency response functions. Eissa et al. [16] investigated the effects of saturation phenomena on non-linear oscillating systems subject to multi-parametric and/or external excitations. They reported the occurrence of saturation phenomena at different parameters values. Eissa and Sayed [17-19] and Sayed [20], studied the effects of different active controllers on simple and spring pendulum at the primary resonance via negative velocity feedback or its square or cubic. Amer et al. [21], studied the dynamical system of a twin-tail aircraft, which is described by two coupled second order nonlinear differential equations having both quadratic and cubic nonlinearities, solved and controlled. Sayed and Kamel [22, 23] investigated the effect of different controllers on the vibrating system and the saturation control of a linear absorber to reduce vibrations due to rotor blade flapping motion. Sayed et al. [24] investigated the non-linear dynamics of a two-degree-of freedom vibration system including quadratic and cubic non-linearities subjected to external and parametric excitation forces.

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2 Mathematical modeling

The proposed modified model [14, 15] governing equations of the considered dynamical system under investigation are given by equations:

$$\ddot{u}_1+\omega_1^2 u_1+(\mu_1+\alpha)\dot{u}_1+\alpha_1 u_1^2+\alpha_2 u_1^3+\alpha_3 u_1^5-\beta u_2$$
$$-\alpha\dot{u}_2=F_1\cos(\Omega_1 t)+u_1 F_2\cos(\Omega_2 t) \quad (1)$$

$$\ddot{u}_2+\omega_2^2 u_2+(\mu_2+\mu\alpha)\dot{u}_2+\alpha_4 u_2^2$$
$$=\mu\beta u_1(t-\tau_1)+\mu\alpha\dot{u}_1(t-\tau_2) \quad (2)$$

where $u_1,\dot{u}_1,\ddot{u}_1$ are the displacement, velocity and acceleration of the beam respectively and $u_2,\dot{u}_2,\ddot{u}_2$ are the displacement, velocity and acceleration of the absorber, μ_1 and μ_2 are the non-dimensionless damping coefficients of the two modes respectively, α and β are the non-dimensionless control gain parameters, ω_1,ω_2 are natural frequencies and F_1, F_2 are the external and parametric excitation forces respectively and Ω_1, Ω_2 are the external and parametric frequencies respectively, $\alpha_1,\alpha_2,\alpha_3$ and α_4,μ are the other characteristic coefficient of the structure and control respectively, τ_1 and τ_2 are the time delay for the displacement and velocity feedback of the system respectively.

2.1. Perturbation Analysis

Multiple scale perturbation method [25] is conducted to obtain first order approximate solutions for Eqs. (1)-(2). Assuming the solution in the form:

$$u_1(t;\varepsilon)=u_{10}(T_0,T_1)+\varepsilon u_{11}(T_0,T_1) \quad (3)$$

$$u_2(t;\varepsilon)=u_{20}(T_0,T_1)+\varepsilon u_{21}(T_0,T_1) \quad (4)$$

The time derivatives are given by:

$$\frac{d}{dt}\equiv D_0+\varepsilon D_1\ , \qquad \frac{d^2}{dt^2}\equiv D_0^2+2\varepsilon D_0 D_1 \quad (5)$$

where $T_n=\varepsilon^n t\ \ (n=0,1)$. T_0 and T_1 are the fast and slow time scales respectively. To make damping, nonlinearities, primary resonance force, principle parametric resonance force and controller parameters appear in the same perturbation equations we scale the equation parameters as:

$$\mu_1=\varepsilon\hat{\mu}_1,\ \mu_2=\varepsilon\hat{\mu}_2, \alpha_n=\varepsilon\hat{\alpha}_n,\ \beta=\varepsilon\hat{\beta}, \alpha=\varepsilon\hat{\alpha},$$
$$F_1=\varepsilon\hat{F}_1, F_2=\varepsilon\hat{F}_2,\ n=1,2,3,4 \quad (6)$$

Substituting Eqs. (3)-(6) into Eqs. (1)-(2) and equating the coefficients of the same power of ε in both sides, we obtain:

$O(\varepsilon^0)$:

$$(D_0^2+\omega_1^2)u_{10}=0 \quad (7)$$

$$(D_0^2+\omega_2^2)u_{20}=0 \quad (8)$$

$O(\varepsilon^1)$:

$$(D_0^2+\omega_1^2)u_{11}=-2D_0 D_1 u_{10}-(\hat{\mu}_1+\hat{\alpha})D_0 u_{10}-\hat{\alpha}_1 u_{10}^2$$
$$-\hat{\alpha}_2 u_{10}^3-\hat{\alpha}_3 u_{10}^5+\hat{\beta}u_{20}+\hat{\alpha}D_0 u_{20}$$
$$+\hat{F}_1\cos(\Omega_1 T_0)+u_{10}\hat{F}_2\cos(\Omega_2 T_0) \quad (9)$$

$$(D_0^2+\omega_2^2)u_{21}=-2D_0 D_1 u_{20}-(\hat{\mu}_2+\mu\hat{\alpha})D_0 u_{20}$$
$$-\hat{\alpha}_4 u_{20}^2+\mu\hat{\beta}u_{10\tau_1}+\mu\hat{\alpha}D_0 u_{10\tau_2} \quad (10)$$

The solution of Eqs. (7) and (8) can be expressed in the form:

$$u_{10}=A_1\exp(i\,\omega_1 T_0)+cc \quad (11)$$

$$u_{20}=A_2\exp(i\,\omega_2 T_0)+cc \quad (12)$$

where A_1 and A_2 are a complex functions in T_1 and cc indicates the complex conjugates of the preceding terms. Substituting Eqs. (11)-(12) into Eqs. (9)-(10) and after eliminating the secular terms, the non-homogeneous solutions of Eqs. (9)-(10) are:

$$u_{11}=E_1\exp(2i\,\omega_1 T_0)+E_2\exp(3i\,\omega_1 T_0)+E_3\exp(5i\,\omega_1 T_0)$$
$$+E_4\exp(i\,\omega_2 T_0)+E_5\exp(i\,\Omega_1 T_0)+E_6\exp(i\,(\Omega_2+\omega_1)T_0)$$
$$+E_7\exp(i\,(\Omega_2-\omega_1)T_0)+E_8+cc \quad (13)$$

$$u_{21}=E_9\exp(i\,\omega_1(T_0-\tau_1))+E_{10}\exp(i\,\omega_1(T_0-\tau_2))$$
$$+E_{11}\exp(2i\,\omega_2 T_0)+E_{12}+cc \quad (14)$$

where $E_i\ \ (i=1,2,\ldots,12)$ are complex functions in T_1. From the above derived solutions, the reported resonance cases are:

Primary resonance: $\Omega_1\cong\omega_1$.

Sub-harmonic resonance: $\Omega_2\cong 2\,\omega_1$.

Internal or secondary resonance: $\omega_1\cong\omega_2$.

Simultaneous or incident resonance

Any combination of the above resonance cases is considered as simultaneous resonance.

3. Stability of the System

Investigating numerically these resonance cases showed that the worst one is the simultaneous primary and principle parametric in the presence of internal resonance case, which is given by $\Omega_1\cong\omega_1$, $\Omega_2\cong 2\omega_1$ and $\omega_1\cong\omega_2$. Introducing the external and internal detuning parameters σ_1, σ_2 and σ_3 to convert the small-divisor terms into the secular terms, according to:

$$\Omega_1=\omega_1+\sigma_1=\omega_1+\varepsilon\hat{\sigma}_1,\ \ \Omega_2=2\omega_1+\sigma_2=2\omega_1+\varepsilon\hat{\sigma}_2,$$
$$\omega_1=\omega_2+\sigma_3=\omega_2+\varepsilon\hat{\sigma}_3 \quad (15)$$

Substituting Eq. (15) into Eqs. (9)-(10) and eliminating the secular terms, leads to the solvability conditions :

$$2i\,\omega_1 D_1 A_1=-i\,\omega_1(\hat{\mu}_1+\hat{\alpha})A_1-3\hat{\alpha}_2 A_1^2\bar{A}_1-10\hat{\alpha}_3 A_1^3\bar{A}_1^2$$
$$+(\hat{\beta}+i\,\omega_2\hat{\alpha})A_2\exp(-i\,\hat{\sigma}_3 T_1)+(\hat{F}_1/2)\exp(i\,\hat{\sigma}_1 T_1)$$

ISSN 2229-5518

$$+(\hat{F}_2\bar{A}_1/2)\exp(i\,\hat{\sigma}_2 T_1) \qquad (16)$$

$$2i\,\omega_2 D_1 A_2 = -i\,\omega_2(\hat{\mu}_2+\mu\hat{\alpha})A_2 + \mu\hat{\beta}A_1\exp(i\,(\hat{\sigma}_3 T_1-\omega_1\tau_1)) + i\,\mu\hat{\alpha}\omega_1 A_1\exp(i\,(\hat{\sigma}_3 T_1-\omega_1\tau_2)) \qquad (17)$$

Using the polar form

$$A_n = (a_n/2)e^{i\gamma_n} \qquad (n=1,2) \qquad (18)$$

where a_n and γ_n are the steady state amplitudes and phases of the motion respectively. Substituting Eq. (18) into Eqs. (16) and (17) and separating real and imaginary parts yields.
Then it follows that the steady state solutions are given by

$$-\frac{(\mu_1+\alpha)}{2}a_1 - \frac{\beta}{2\omega_1}a_2\sin\theta_3 + \frac{\omega_2\alpha}{2\omega_1}a_2\cos\theta_3 + \frac{F_1}{2\omega_1}\sin\theta_1 + \frac{F_2}{4\omega_1}a_1\sin\theta_2 = 0 \qquad (19)$$

$$a_1\sigma - \frac{3\alpha_2}{8\omega_1}a_1^3 - \frac{5\alpha_3}{16\omega_1}a_1^5 + \frac{\beta}{2\omega_1}a_2\cos\theta_3 + \frac{\omega_2\alpha}{2\omega_1}a_2\sin\theta_3 + \frac{F_1}{2\omega_1}\cos\theta_1 + \frac{F_2}{4\omega_1}a_1\cos\theta_2 = 0 \qquad (20)$$

$$-\frac{(\mu_2+\mu\alpha)}{2}a_2 + \frac{\mu\alpha\omega_1}{2\omega_2}a_1\cos(\theta_3-\omega_1\tau_2) + \frac{\mu\beta}{2\omega_2}a_1\sin(\theta_3-\omega_1\tau_1) = 0 \qquad (21)$$

$$a_2(\sigma+\sigma_3) + \frac{\mu\beta}{2\omega_2}a_1\cos(\theta_3-\omega_1\tau_1) - \frac{\mu\alpha\omega_1}{2\omega_2}a_1\sin(\theta_3-\omega_1\tau_2) = 0 \qquad (22)$$

4. Results and Discussions

Results are presented in graphical forms as steady state amplitudes against detuning parameters and as time history or the response for both structure and controller. Fig. 1 shows that the steady state amplitude of the structure without controller at simultaneous primary and principle parametric resonance where $\Omega_1 \cong \omega_1,\ \Omega_2 \cong 2\omega_1$ is about twelve times that of the maximum excitation amplitude F_1, the solution is stable with multi-limit cycle.

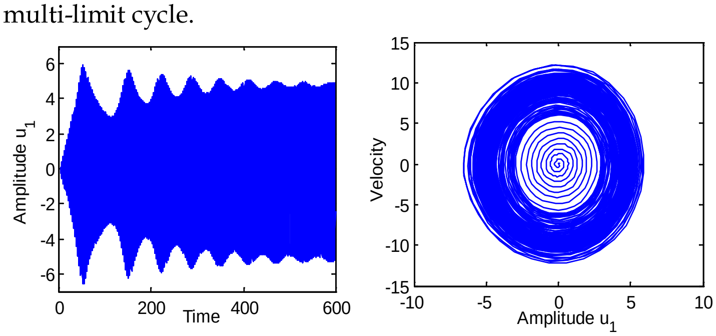

Fig. 1. System behavior without controller at simultaneous primary and principle parametric resonance $\Omega_1 \cong \omega_1,\ \Omega_2 \cong 2\omega_1$.

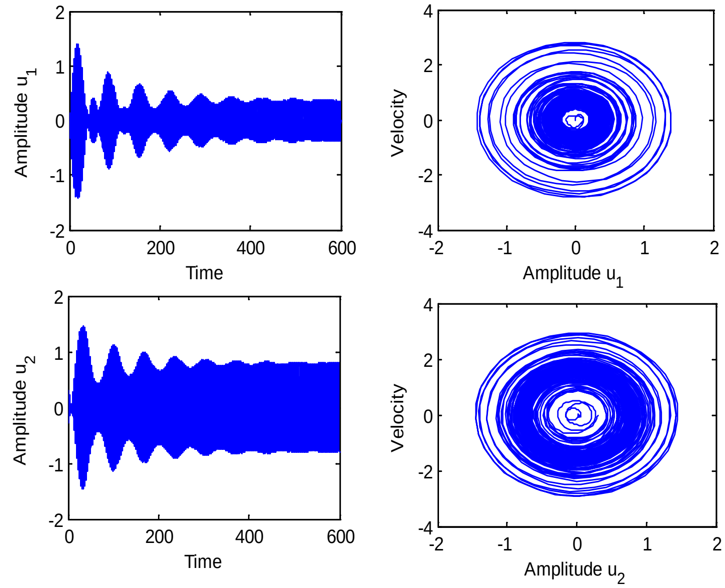

Fig.2. System behavior with controller at resonance $\Omega_1 \cong \omega_1, \Omega_2 \cong 2\omega_1, \omega_1 \cong \omega_2$.

$\mu_1 = 0.009,\ \alpha = 0.0001,\ \alpha_1 = 0.08,\ \alpha_2 = 0.013,$
$\alpha_3 = -0.0008, F_1 = 0.4, F_2 = 0.2, \omega_1 = 2,$
$\mu_2 = 0.07, \alpha_4 = 0.02, \mu = 0.6, \beta = 0.5,$

Fig. 2 shows that the steady state amplitude of the system with absorber at the simultaneous resonance $\Omega_1 \cong \omega_1, \Omega_2 \cong 2\omega_1$ and $\omega_2 \cong \omega_1$. It can be seen for the main system that the steady state amplitude is 0.94%, but the steady state amplitude of the controller is about 200% of excitation amplitude F_1. This means that the effectiveness of the controller E_a (E_a = the steady state amplitude of the main system without absorber/ the steady state amplitude of main system with absorber) is about 14. Also, the oscillations of the system and controller have multi-limit cycle and limit cycle respectively.

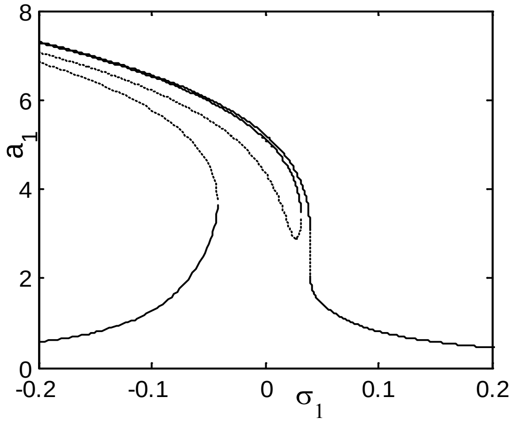

Fig.3. Effects of the detuning parameter σ_1.

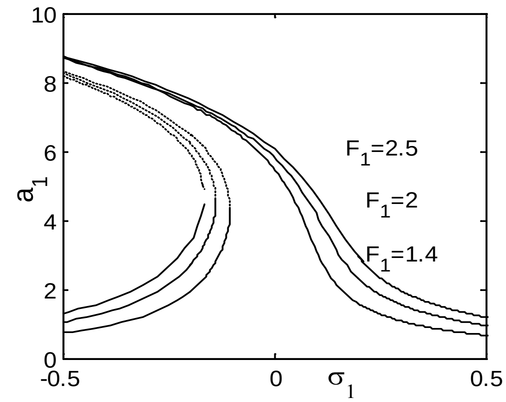

Fig. 5. Effects of the excitation amplitude F_1.

Fig.3, shows that the steady state amplitudes of the system against the detuning parameter σ_1 as a basic case. In this figure, the response amplitude consists of a continuous curve which is bent to the left and has softening spring type and there exists jump phenomena. This continuous curve has stable and unstable solutions. For positive and negative value of the nonlinear parameter, α_3, the curve is bent to right or left leading to the occurrence of the jump phenomena and multi-valued amplitudes produce either soft or hard spring respectively as shown in Fig. 4. Fig. 5, shows that the steady state amplitude of the system is a monotonic increasing function in the excitation amplitude F_1.

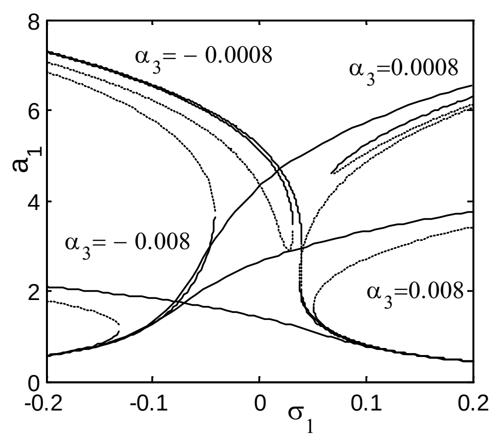

Fig. 4. Effects of the nonlinear parameter α_3.

4.1. Comparison Study

In the previous work [14, 15], studied the same system when subjected to multi-external [14] or multi-parametric [15].

In our study, the response and stability of the system under external and parametric excitation forces are investigated using the multiple time scale method. The second-order approximation is obtained to consider the influence of the quadratic and cubic terms on non-linear dynamic characteristics of the system. All possible resonance cases are extracted and investigated at this approximation order. The case of simultaneous primary and principle parametric resonance in the presence of 1:1 internal resonance is considered. The stability of the system is investigated using both frequency response equations and phase-plane method. It is quite clear that some of the simultaneous primary resonance cases are undesirable in the design of such system as they represent some of the worst behavior of the system.

5. Conclusions

The nonlinear response of a system subjected to external and parametric excitations have been studied. The problem is described by a two-degree-of-freedom system of nonlinear ordinary differential equations. The case of simultaneous primary and principle parametric resonance in the presence of one-to-one internal resonance is studied by applying multiple time scale perturbation method using a second-order approximation. Both the frequency response equations and the phase-plane technique are applied to study the stability of the system. The effect of the different parameters of the system is studied numerically. From the above study the following may be concluded:

1- The simultaneous resonance case $\Omega_1 \cong \omega_1$, $\Omega_2 \cong 2\omega_1$ is the worst cases and it should be avoided in design.

2- For positive and negative values of the nonlinear parame-

ISSN 2229-5518

ters α_3, the curves are bent to right or left leading to the occurrence of the jump phenomena and multi-valued amplitudes produce either hard or soft spring respectively.

3-The steady state amplitude of the system are a monotonic increasing function in the excitation amplitude F_1.

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